FAST ELECTROMAGNETIC ANALYSIS OF DENSE SHIELDED INTEGRATED CIRCUITS USING THE ADAPTIVE INTEGRAL METHOD (AIM)

Vladimir I. Okhmatovski and Andreas C. Cangellaris

Department of Electrical & Computer Engineering, University of Illinois at Urbana-Champaign, 1406 West Green Street, Urbana, IL 61801, U.S.A.

***Abstract* — The adaptive integral method (AIM) is used to accelerate the electromagnetic solution of dense integrated circuits inside metallic enclosures of rectangular cross section. The computational complexity and memory requirements for the proposed AIM-based electromagnetic solver scale as O(NlogN) and O(N), respectively, where N is the number of unknowns in the discrete approximation of the governing integral equation. The accuracy and efficiency of the solver is demonstrated through its application to the modeling of a shielded patch array used for spatial power combining.**

I. INTRODUCTION

Dense planar and quasi-planar integrated circuits are becoming the norm in the development of multi-layered packages and substrates for the realization of compact, system-in-a-package (SIP) and system-on-a-chip (SOC) multi-functional designs. The interconnection density in the multi-layered substrates for such systems is so high that the number of unknowns in the matrix approximation of the governing electromagnetic equations for these problems is in the order of tens or even hundreds of thousands. Direct factorization of these matrices is computationally prohibitive. Furthermore, the iterative solution using state-of-the-art conjugate gradient methods becomes unattractive when the number of unknowns, N, is in the order of tens of thousands, since both memory requirements for the storage of the method-of-moments (MoM) matrix and the complexity of the matrix-vector products involved in the iteration scale as $O(N^2)$. Another class of electromagnetic problems inside rectangular waveguides that is hindered by excessive memory and computational complexity requirements is the class of the integrated, passive, planar and quasi-planar circuitry used in waveguide-based, spatial power combining systems [1]. In addition to high density, these systems tend to be electrically large, spanning several wavelengths in each direction. Consequently, the number of unknowns involved in their MoM solution becomes once again prohibitively large.

To address the aforementioned complexity and establish a computationally efficient approach to the electromagnetic characterization of such high-density and/or electrically large planar and quasi-planar circuits, the AIM methodology of [2] is extended to the case where the electromagnetic structure of interest is placed inside a rectangular metallic waveguide. The computational complexity and memory requirements of the developed AIM-based iterative MoM solver scale as $O(N \log N)$ and $O(N)$, respectively, thus enabling the expedient solution of very dense and electrically large shielded integrated circuits.

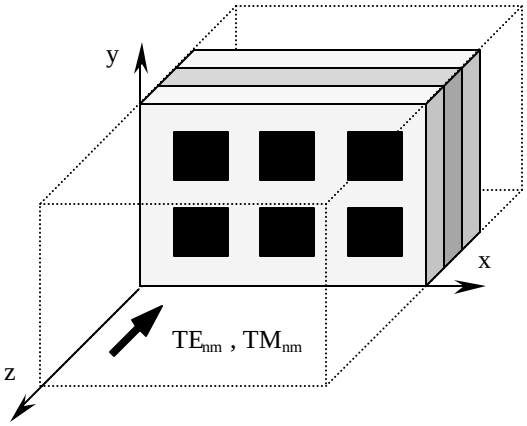

Fig. 1. Generic geometry of a planar microstrip structure inside a metallic rectangular waveguide.

II. MODIFIED AIM FOR WAVEGUIDES

In the following, the fundamental steps in the mathematical development of a modified version of AIM suitable for structures shielded inside metallic, rectangular waveguides is presented. Without loss of generality and for the sake of simplicity in the mathematical formulation it is assumed that the integrated circuit structure is planar, and thus the unknown current densities exhibit only x and y components in the rectangular waveguide. The electromagnetic boundary value problem in the directions

parallel to the cross section of the of interest is cast in integral equation form as follows,

$$\int_S G^{xx}(\vec{r},\vec{r}')\cdot J_x(\vec{r}')ds' + \int_S G^{xy}(\vec{r},\vec{r}')\cdot J_y(\vec{r}')ds' = E_x^{inc}(\vec{r}) \quad (1a)$$

$$\int_S G^{yx}(\vec{r},\vec{r}')\cdot J_x(\vec{r}')ds' + \int_S G^{yy}(\vec{r},\vec{r}')\cdot J_y(\vec{r}')ds' = E_y^{inc}(\vec{r}) \quad (1b)$$

The relevant Green's functions for the case of waveguides with layered dielectric filling are described in detail in [3], [4] and will not be repeated here. Even though the excitation is shown in terms of the incident fields of propagating and evanescent modes in the waveguide, arbitrary excitations in terms of localized sources can be handled also. The MoM approximation of (1) begins with the expansion of the unknown current densities in terms of a set of known basis functions $J_a(x,y) = \sum_{n=1}^{N_a} I_n^{(a)} f_n^{(a)}(x,y)$, where a and b assume the values x and y. Substitution of these expansions in (1), followed by a Galerkin's testing process, leads to the following matrix form of the MoM approximation of the problem [3],

$$\begin{bmatrix} \mathbf{Z}^{xx} & \mathbf{Z}^{xy} \\ \mathbf{Z}^{yx} & \mathbf{Z}^{yy} \end{bmatrix} \begin{bmatrix} \mathbf{I}^x \\ \mathbf{I}^y \end{bmatrix} = \begin{bmatrix} \mathbf{V}^x \\ \mathbf{V}^y \end{bmatrix}, \quad (2)$$

where the elements of the matrix $\mathbf{Z}^{ab}$ are the interactions between the basis functions in the expansions of the a and b components of the unknown current densities. From the remaining terms in (2), the vectors $\mathbf{I}^x$ and $\mathbf{I}^y$ contain the coefficients in the expansions, and the vectors $\mathbf{V}^x$ and $\mathbf{V}^y$ describe the weighted forms of the excitation. For electrically large or dense circuits, iterative methods (e.g. conjugate gradient) are needed to solve (2). The computational complexity of such methods is dominated by the $O(N^2)$ matrix-vector product in each iteration.

AIM addresses this complexity by splitting of the product $\mathbf{Z}\cdot\mathbf{I}$ of MoM matrix and vector of current expansion coefficients into two parts. One part includes the "near-zone" interactions and the remaining includes the "far-zone" interactions. This splitting may be cast in the form,

$$\mathbf{Z}\cdot\mathbf{I} = \mathbf{Z}_{MoM}^{near}\cdot\mathbf{I} + \mathbf{Z}^{far}\cdot\mathbf{I} = \mathbf{Z}_{MoM}^{near}\cdot\mathbf{I} + \left(\mathbf{Z}_{AIM}\cdot\mathbf{I} - \mathbf{Z}_{AIM}^{near}\cdot\mathbf{I}\right). \quad (3)$$

In the above equation, $\mathbf{Z}_{MoM}^{near}\cdot\mathbf{I}$ contains the interactions between closely spaced elements, separated by distances less than some threshold distance. These interactions are calculated using standard MoM. Clearly, the matrix $\mathbf{Z}_{MoM}^{near}$ is very sparse and the computational complexity of $\mathbf{Z}_{MoM}^{near}\cdot\mathbf{I}$ is $O(N)$. The "far-zone" interactions are treated in a very different manner, as explained next. However, before we proceed any further, it is important to clarify that the distinction between "near-zone" and "far-zone" interactions is not based on electrical distance. Thus, the method can be applied with the same effectiveness for both electrically large and sub-wavelength size structures.

First, a rectangular grid is laid over the entire waveguide cross-section. Each of the basis functions, $f_n^{(a)}(x,y)$, in the current expansions is replaced by M^2 equivalent "delta" sources,

$$f_n^{(a)}(\vec{r}) \simeq \hat{f}_n^{(a)}(\vec{r}) = \sum_{m_1=1}^{M}\sum_{m_2=1}^{M} \Lambda_{m_1,m_2,n}^{(a)} d(x - x_{m_1,n}^{(a)}) d(y - y_{m_1,n}^{(a)}), \quad (4)$$

where $\{x_{m_1,n}^{(a)}, y_{m_2,n}^{(a)}\}$ are the locations of the delta sources associated with the nth basis functions in the expansion of the a component of the current density. Various criteria may be used to specify the expansion coefficients $\Lambda^{(a)}$. Our investigation showed that a scheme based on a least square approximation of the waveguide eigen-modes is superior in term of approximation error to the multipole reproduction criteria [2] traditionally used in AIM for open structures. Substitution of (4) into the integrals that define the elements of the MoM matrix yields,

$$Z_{mn}^{ab} = \sum_{k_1=0}^{K_1-1}\sum_{k_2=0}^{K_2-1}\sum_{k_1'=0}^{K_1-1}\sum_{k_2'=0}^{K_2-1} \Lambda_{k_1,k_2,m}^{(a)} G^{ab}\left(k_1\Delta x, k_2\Delta y \mid k_1'\Delta x, k_2'\Delta y\right) \Lambda_{k_1',k_2',n}^{(b)},$$

where $m = 1,\ldots N_a; n = 1,\ldots N_b$. (5)

In the above equation, K_1, K_2 are the total number of the AIM grid points along x and y, respectively. In matrix form, use of the above equivalence helps us rewrite the MoM matrix as follows,

$$\mathbf{Z}^{ab} = \Lambda^{(a)}\cdot\mathbf{G}^{ab}\cdot\Lambda^{(b)^T}. \quad (6)$$

Since only M^2 delta sources are used in (4), the matrix $\Lambda^{(a)}$ has only $M^2\cdot N_a$, $a = x$ or y, non-zero elements. In view of (5), (6), the matrix-vector product in the iterative solution of the MoM equation may be computed as,

$$\mathbf{Z}_{AIM}\cdot\mathbf{I} = \begin{bmatrix} \Lambda^{(x)}\cdot\mathbf{G}^{xx}\cdot\Lambda^{(x)^T}\cdot\mathbf{I}^x + \Lambda^{(x)}\cdot\mathbf{G}^{xy}\cdot\Lambda^{(y)^T}\cdot\mathbf{I}^y \\ \Lambda^{(y)}\cdot\mathbf{G}^{yx}\cdot\Lambda^{(x)^T}\cdot\mathbf{I}^x + \Lambda^{(y)}\cdot\mathbf{G}^{yy}\cdot\Lambda^{(y)^T}\cdot\mathbf{I}^y \end{bmatrix} \quad (7)$$

The complexity of the matrix-vector products in (7) is $O(N\log N)$ because the waveguide Green's function for interactions between the delta sources on the rectangular grid can be written in the form,

$$G^{ab}(k_1,k_2;k_1',k_2') = s_1^{(ab)}\Gamma^{ab}{}_{k_1+k_1',\,k_2-k_2'} + s_2^{(ab)}\Gamma^{ab}{}_{k_1+k_1',\,k_2-k_2'} + s_3^{(ab)}\Gamma^{ab}{}_{k_1-k_1',\,k_2+k_2'} + s_4^{(ab)}\Gamma^{ab}{}_{k_1+k_1',\,k_2+k_2'} \quad (8)$$

where the coefficients $s_i^{(ab)}$, $i = 1,2,3,4$, assume values ± 1 depending on the values of a and b. Due to the presence of the convolution/correlation terms in (8), FFT

can be implemented to reduce the $O(N^2)$ of the matrix-vector products in (7) to $O(N \log N)$. The FFT-based computation of (7) is summarized as follows:

Initialization: Compute Green's function $\Gamma^{ab}{}_{k_1,k_2}$ and its FFTs $F(\Gamma^{ab})$ on the grid $k_{1,2} = 0,\ldots,2(K_{1,2}-1)$ for $a = x, y$ and $b = x, y$.

For each iteration of the iterative matrix solver, do:

Step 1: Compute the products $\Lambda^{(x)^T} \cdot \mathbf{I}^x, \Lambda^{(y)^T} \cdot \mathbf{I}^y$ and their FFTs $F\left\{\Lambda^{(x)^T} \cdot \mathbf{I}^x\right\}$, $F\left\{\Lambda^{(y)^T} \cdot \mathbf{I}^x\right\}$. (Since the matrices $\Lambda^{(a)}$ are very sparse the matrix vector products are of $O(N)$ complexity.)

Step 2: Compute matrices $\mathbf{P}^x$ and $\mathbf{P}^y$ as

$$\begin{aligned}\mathbf{P}^a &= \mathbf{G}^{ax} \cdot \left(\Lambda^{(x)^T} \cdot \mathbf{I}^x\right) + \mathbf{G}^{ay} \cdot \left(\Lambda^{(y)^T} \cdot \mathbf{I}^y\right) \\ &= F^{-1}\left\{\sum_{n=1}^{4} s_n^{(ax)} F_n\left\{\Gamma^{ax}\right\} F_n\left\{\Lambda^{(x)^T} \cdot \mathbf{I}^x\right\} + \right. \\ &\quad \left. + \sum_{n=1}^{4} s_n^{(ay)} F_n\left\{\Gamma^{ay}\right\} F_n\left\{\Lambda^{(y)^T} \cdot \mathbf{I}^y\right\}\right\}\end{aligned} \tag{9}$$

where $F^{-1}\{\ \}$ denotes operation of inverse FFT. In (9) we assigned subscript n to FFT operators $F_n\{\ \}$ to show that a certain rearrangement of indices [5] is required in addition to FFT in order to compute different convolution/correlation terms.

Step 4: Compute the products $\Lambda^{(x)} \cdot \mathbf{P}^x, \Lambda^{(y)^T} \cdot \mathbf{P}^y$, which are of $O(N)$ complexity since $\Lambda^{(x)}$ and $\Lambda^{(y)}$ are sparse.

It is straightforward to show that, for an iterative matrix solution with number of iterations N_{iter} the number of required operations scales as $4N_{iter}a\,N\log N$, where $a\,N\log N$ is time per FFT operation.

At each iteration, the above process calculates the product $\mathbf{Z}_{AIM} \cdot \mathbf{I}$ (see (3)). While the calculation of the "far-zone" interactions using the AIM process is accurate, the calculated "near-zone" interactions are not. Thus, the operation $\mathbf{Z}_{AIM} \cdot \mathbf{I} - \mathbf{Z}^{near}_{AIM} \cdot \mathbf{I}$ is required to correct the calculation of the "near-zone" interactions by replacing the AIM calculated ones with those obtained using the "exact" MoM representation of the expansion function interactions (see (3)). These operations are of $O(N)$ complexity.

In addition to significant reduction in the computational complexity of the iterative solution, an AIM implementation helps to relax memory requirements. Since only the MoM matrix elements that describe the "near-zone" interactions need to be stored, the $O(N^2)$ overhead associated with the storage of the MoM matrix is avoided. Instead, storing of the "near-zone" interactions results in memory requirements of $O(N)$. Furthermore, due to the Toeplitz/Hankel-like character of the Green's function matrices on the AIM grid the memory requirements for them scale also as $O(N)$.

For the benefits of AIM to be meaningful, the "far-zone" interactions need be calculated with sufficient accuracy. This, in turn, is critically dependent on the accuracy of the equivalence defined in (4).

IV. Numerical Results

To demonstrate the performance of the proposed AIM scheme for shielded structures we considered a six patch antenna array embedded in a rectangular waveguide. The problem geometry is shown on Fig. 1. The wide and narrow

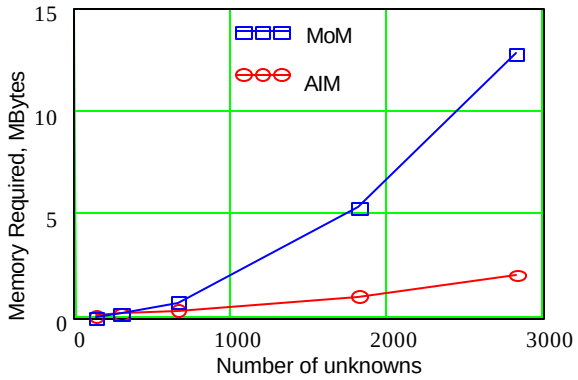

Fig.2 Comparison of memory requirements for MoM and AIM

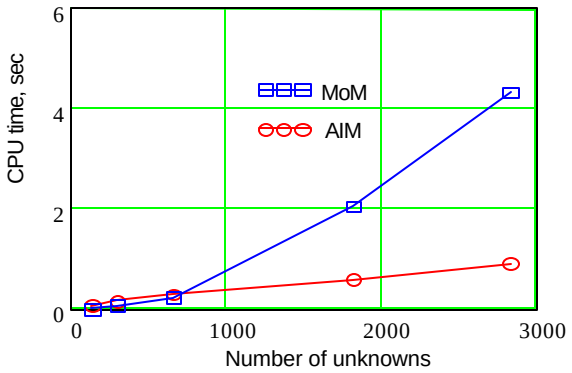

Fig.3 Computational complexity vs. number of unknowns

walls of the waveguide are 2.286 and 1.0287 cm respectively. Patches are printed on a single layer dielectric slab of permittivity $e_r = 2.33$ and thickness $\ell = 2.5\text{cm}$. A detailed description of the remaining parameters of antenna can be found in [3].

In Fig. 2 memory storage required by MoM and AIM versus number of unknowns is shown. One can see that the memory used by AIM grows linearly with the number of unknowns *N*, while the memory required for a standard MoM solution grows as N^2. Such dramatic memory savings are possible because storage of the entire matrix $\mathbf{Z}$ is avoided in the AIM solution. Also it is noted that

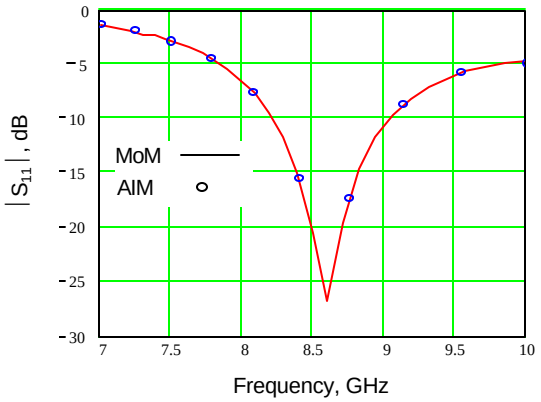

Fig.4 Magnitude of S_{11} for the 6 patch antenna embedded in a rectangular waveguide

only a few elements of $\mathbf{Z}$, the ones associated with near interactions, need to be computed. Hence the AIM scheme fills the matrix in *O(N)* CPU operations instead of $O(N^2)$ operations traditionally required by MoM.

In Fig. 3 we demonstrate that AIM leads to *O(N log N)* CPU time complexity per iteration for the matrix-vector product $\mathbf{Z} \cdot \mathbf{I}$, compared to $O(N^2)$ complexity associated with the method of moments. Clearly, for a relatively low number of unknowns direct matrix-vector evaluation is faster. Time saving advantages for the iterative matrix solver in AIM implementation only becomes obvious when number of unknowns exceeds 10^3.

Finally, to validate the formulation we computed the current distribution on the patches of the antenna structure and the associated S-parameters using both MoM and AIM in the frequency range from 7 to 10 GHz. As shown in Fig. 4 excellent agreement is observed between MoM and AIM solutions.

V. Conclusion

Implementation of fast techniques such as the adaptive integral method (AIM) becomes inevitable when the number of unknowns associated with a scattering problem is large. In this work we demonstrated how AIM can be utilized for the computationally efficient solution of planar structures shielded inside metallic waveguides.

Acknowledgement

This work is supported by an Army Research Office – MURI grant, under the Spatial and Quasi-Optical Power Combining DAAG-55-97-0132.

References

[1] A. B. Yakovlev, S. Ortiz, M. Ozkar, A. Mortazawi, and M. B. Steer, "Electromagnetic modeling and experimental verification of a complete waveguide-based aperture-coupled patch amplifier array," *2000 IEEE MTT-S Int. Microwave Symp. Dig.*, vol. 2, pp. 801-804, June 2000.

[2] E. Bleszynski, M. Bleszynski, and T. Jaroszewicz, "AIM: Adaptive integral method for solving large-scale electromagnetic scattering and radiation problems," *Radio Science*, vol. 31, no. 5, pp. 1225-1251, 1996.

[3] A. I. Khalil, A. B. Yakovlev, and M. B. Steer, "Efficient Method of Moments formulation for the modeling of planar conductive layers in a shielded guided-wave structure," *IEEE Trans. Microwave Theory Tech.*, vol. 47, no. 9, pp. 1730-1736, Sept. 1999.

[4] M.-J. Park, and S. Nam, "Rapid calculation of the Green's function in the shielded planar structures," *IEEE Microwave and guided wave letters*, vol. 7, no. 10, pp. 326-328, Oct. 1997.

[5] W. Press, S. Teukolsky, W. Vetterling, and B. Flannery, "Numerical recipes in Fortran 77," *Cambridge University Press*, 1999.